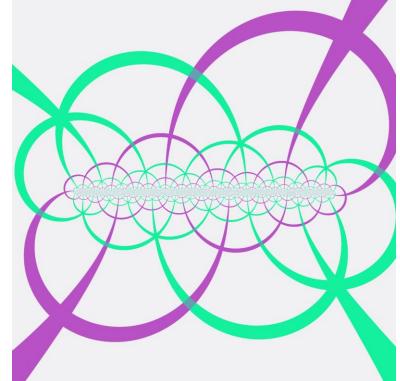
Adventures with the GPU

Roice Nelson

GE Aviation, Austin TX

My goals for this talk

- Provide resources and motivation to get started with shader programming roice3.org/icerm
- A few mathematical detours
- Share fun with @TilingBot and a resulting art piece

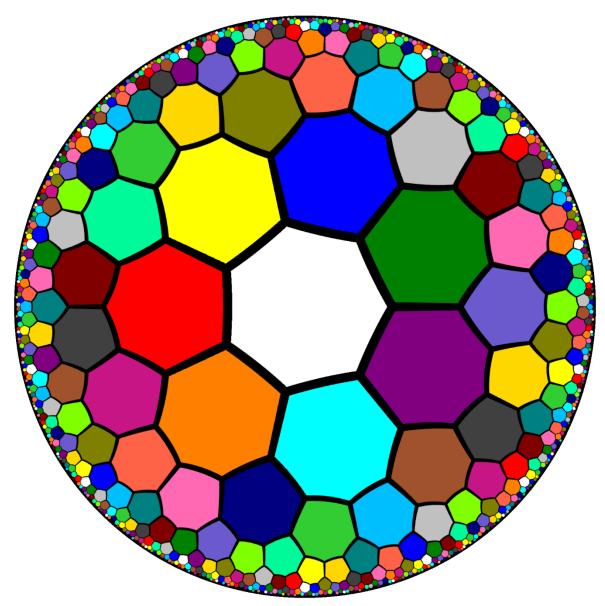


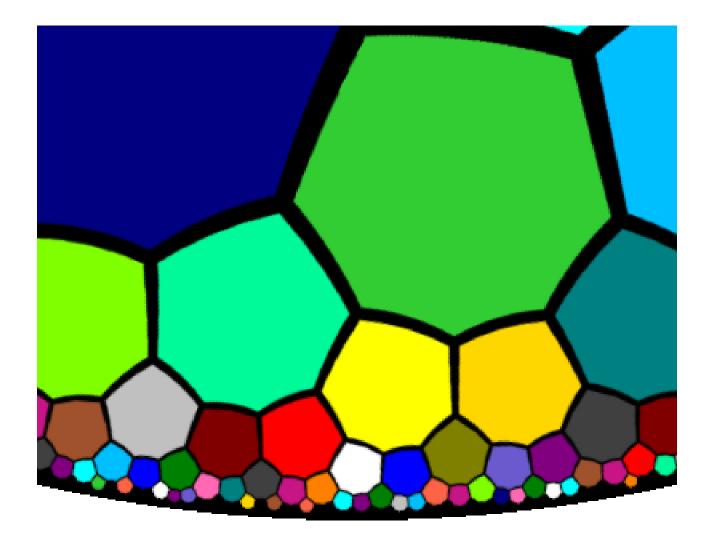
• Tons and tons of pictures and animations! Maybe too many

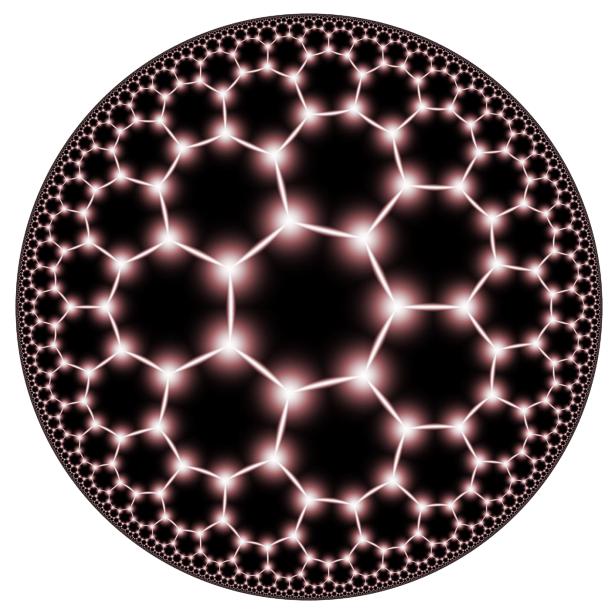
What is a shader?

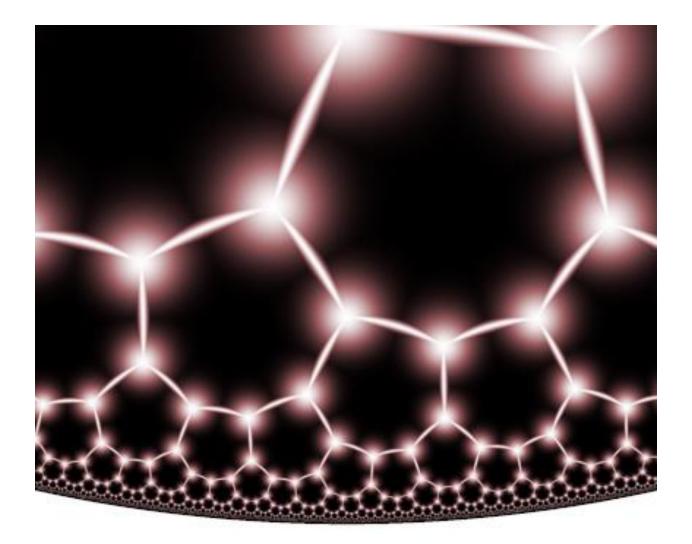
Shaders are little programs that run on the GPU. These programs run at certain points of the graphics pipeline.

```
void mainImage( out vec4 fragColor, in vec2 fragCoord )
{
    // Normalized pixel coordinates (from 0 to 1)
    vec2 uv = fragCoord/iResolution.xy;
    // Time varying pixel color
    vec3 col = 0.5 + 0.5*cos(iTime+uv.xyx+vec3(0,2,4));
    // Output to screen
    fragColor = vec4(col,1.0);
}
```

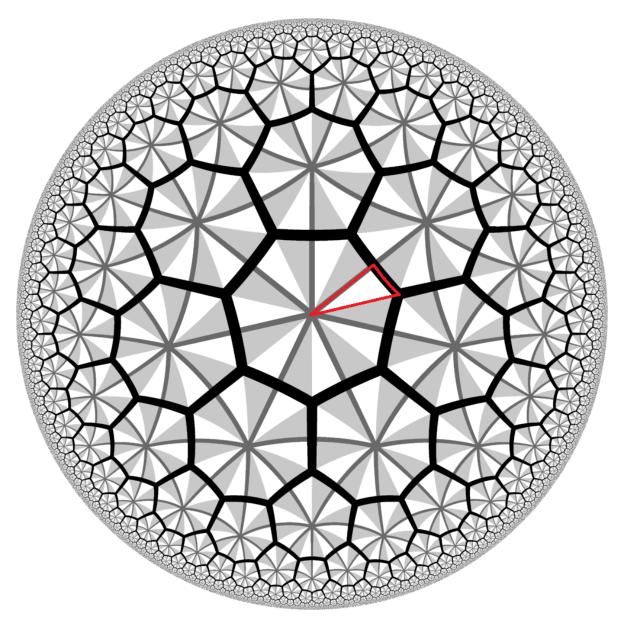




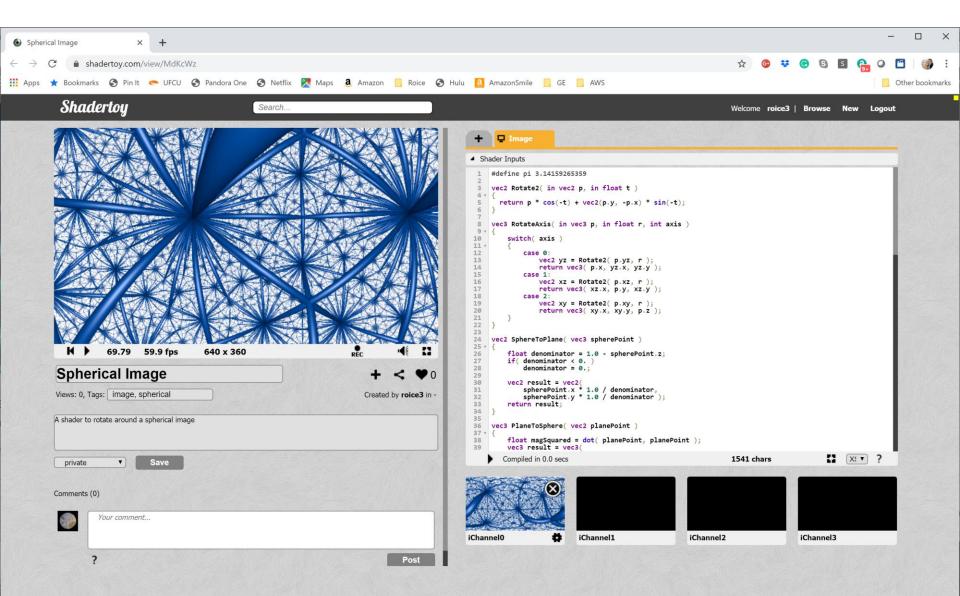




"Folding"



www.shadertoy.com



Shader #2: Isometry classes of hyperbolic space

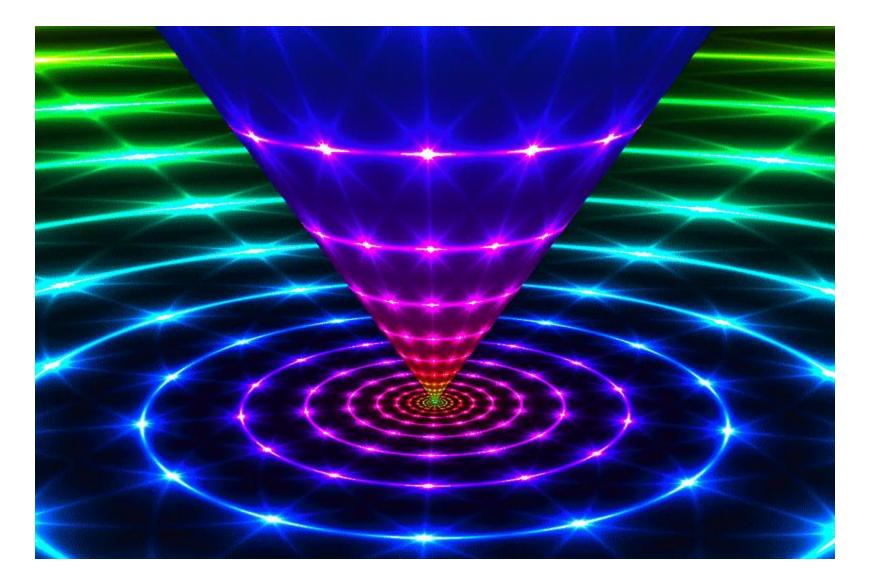
$$F(z) = \frac{az+b}{cz+d}$$

$$\widehat{\boldsymbol{\mathcal{C}}} = \boldsymbol{\mathcal{C}} \cup \{\infty\}$$

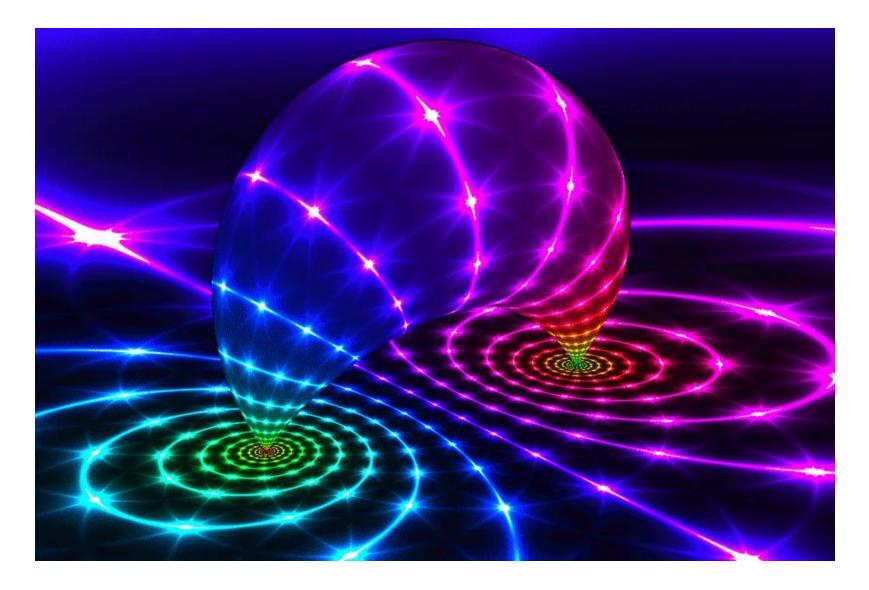
Group of Möbius Transformations

 $PSL(2, \mathbf{C}) \cong PGL(2, \mathbf{C})$

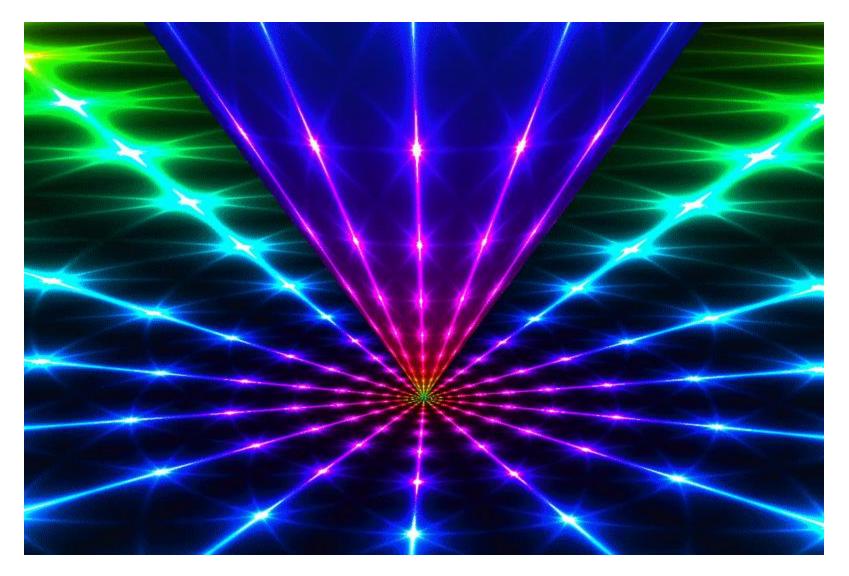
This is not a cone



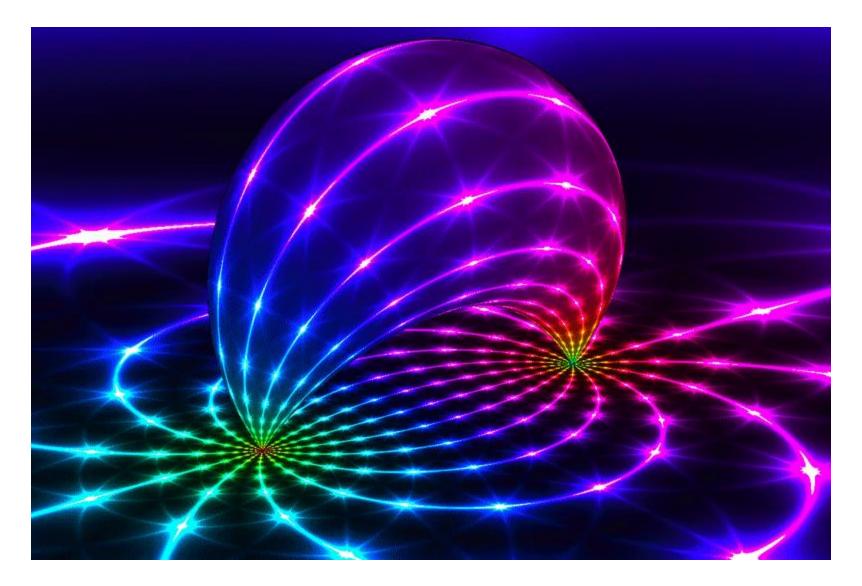
It's a cylinder in UHS model: Elliptic Isometry



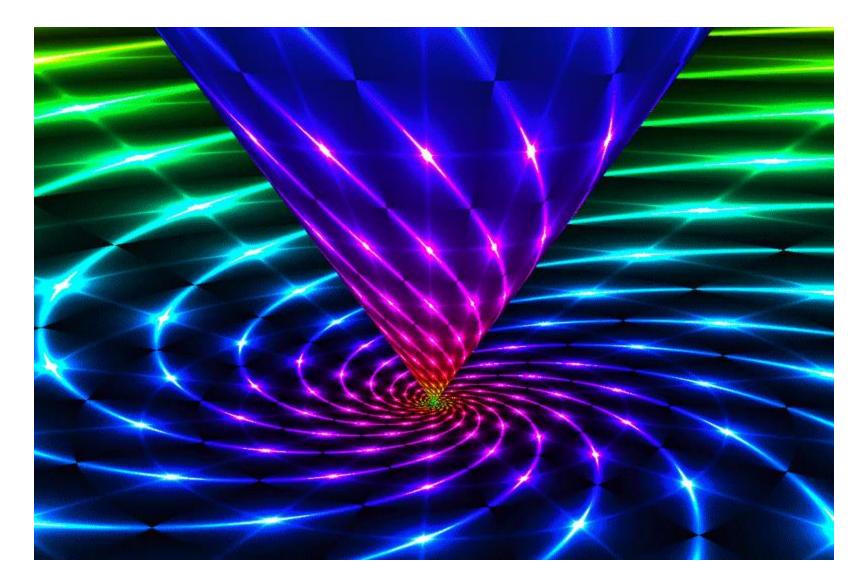
Hyperbolic Isometry



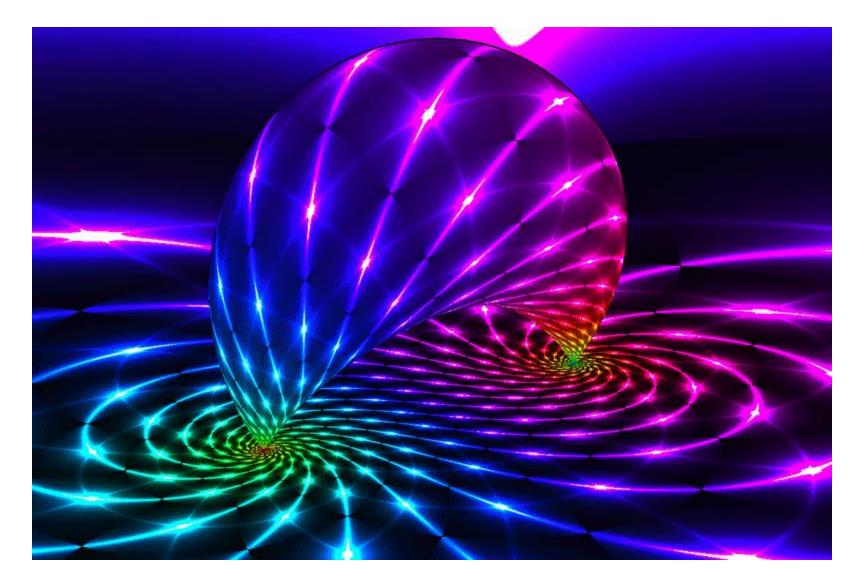
Hyperbolic Isometry



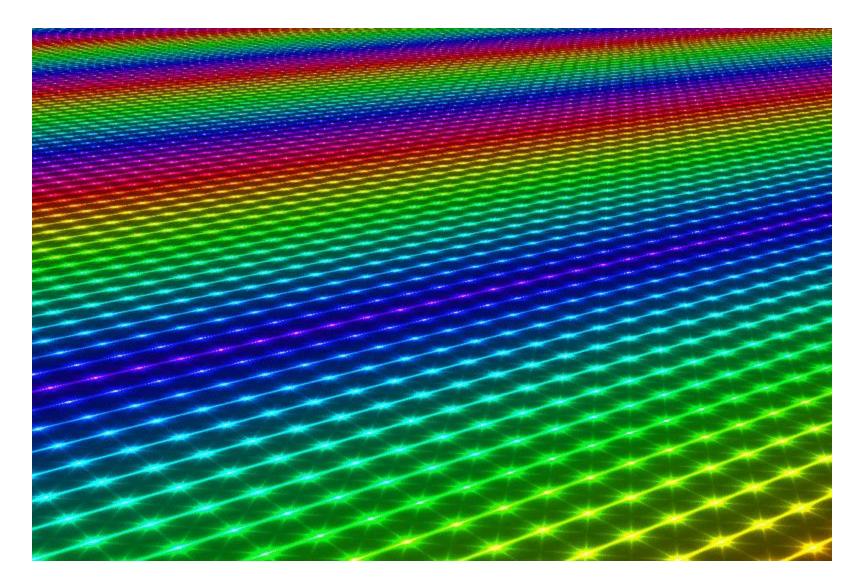
Loxodromic Isometry



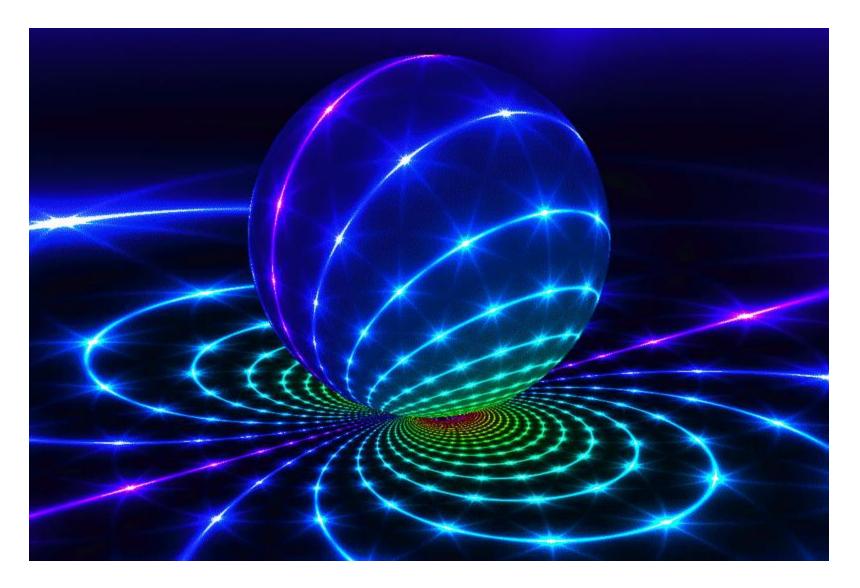
Loxodromic Isometry



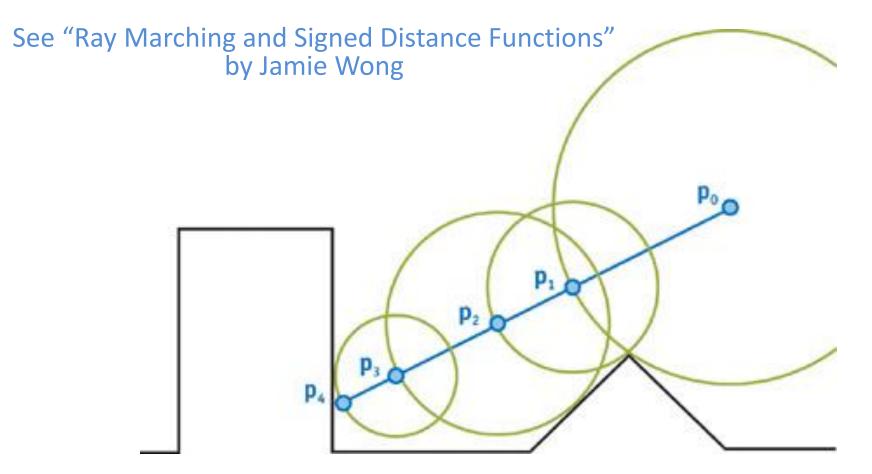
This is not a plane



It's a horosphere: Parabolic Isometry



Raymarching



Credit: GPU Gems 2: Chapter 8

Quaternions!

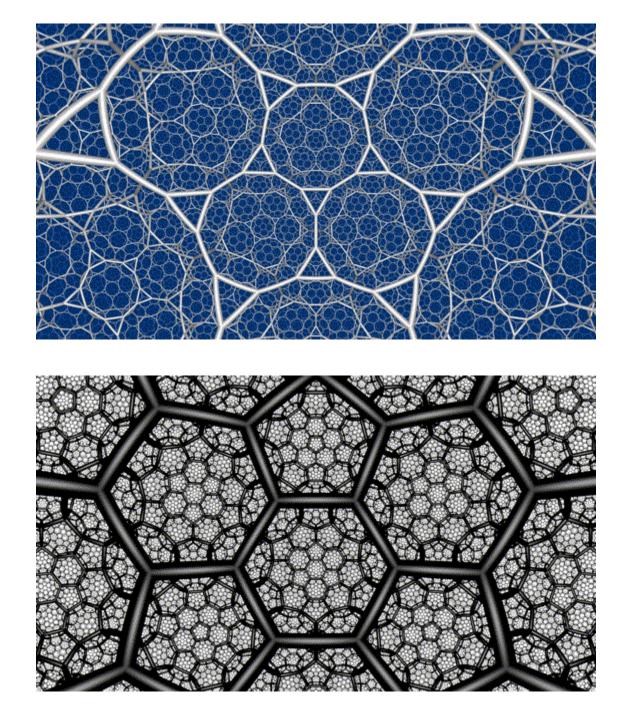
$$z \mapsto \frac{az+b}{cz+d}$$
, $z \in \widehat{C}$

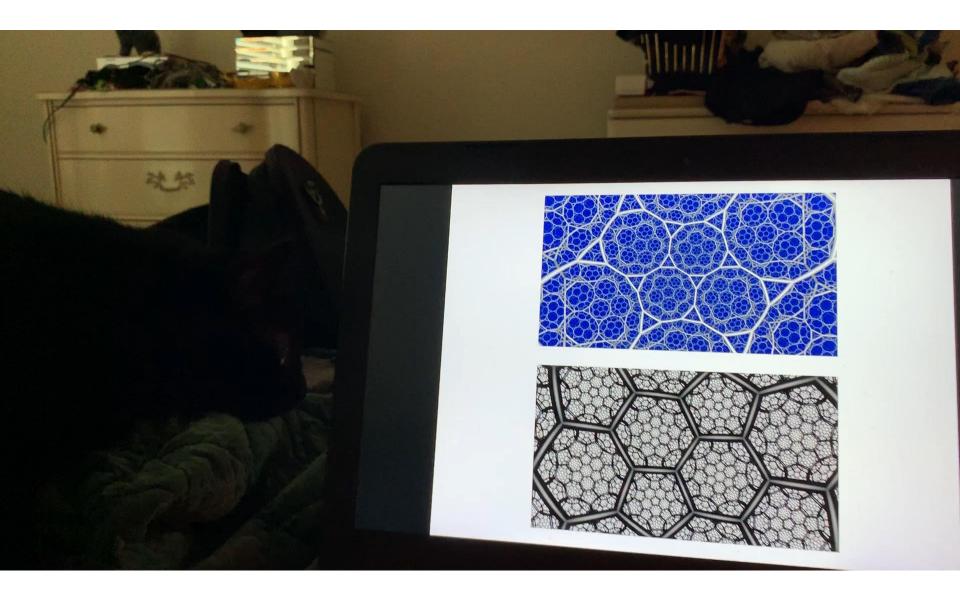
$$w = z + y \boldsymbol{j}$$
 , $y \in \boldsymbol{R}^+$

$$w \mapsto \frac{aw+b}{cw+d}$$

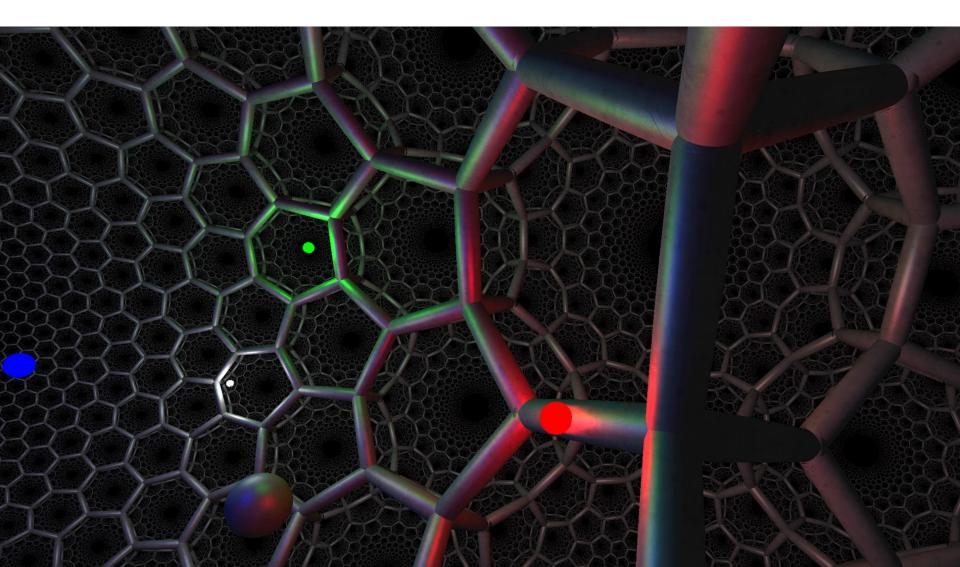
Shader #3: Spherical Images





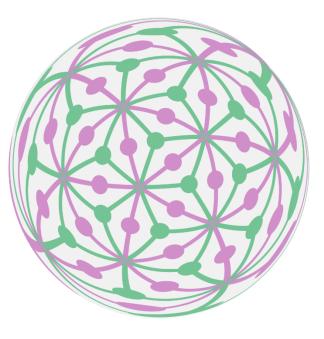


Shader #4: Hyperbolic VR using Raymarching Folding AND Raymarching, see Henry's NSF video!



Utilities

- Shadertoy-render
- ffmpeg
- Pov-Ray
- LinqToTwitter



Again, links (and scripts) at: roice3.org/icerm

In my experience...

Advantages

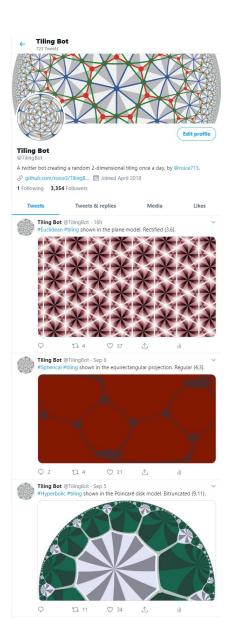
- Fast!
- Motion
- Quality
- Fractals
- WebGL
- Lots of Examples

Disadvantages

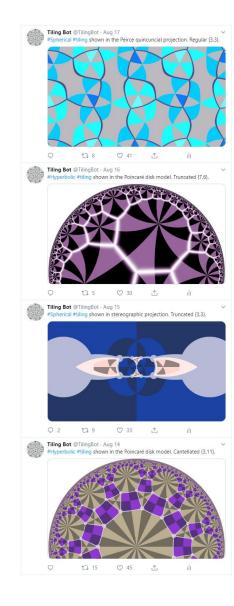
- Hardware
- Debugging
- Optimization
- Low-level
- Code libraries

"The explorer who will not come back or send back his ships to tell his tale is not an explorer, only an adventurer."

-Ursula K. Le Guin, The Dispossessed: An Ambiguous Utopia

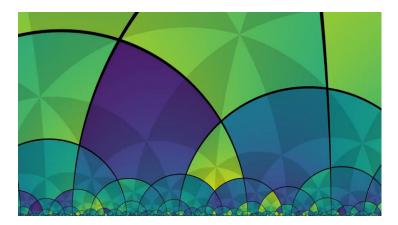


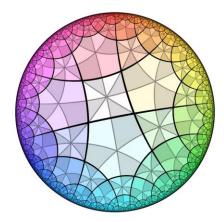
Tiling Bot @TilingBot · Sep 3 #Hyperbolic #tiling shown in the Poincaré disk model. Omnitruncated (14,3). t] 13 0 ♡ 63 ≏ Tiling Bot @TilingBot · Sep 2 #Hyperbolic #tiling shown in the Joukowsky projection. Catalan tiling dual to truncated (9,3). 0 67 Q 1 17 16 1 di. Tiling Bot @TilingBot · Sep 1 #Hyperbolic #tiling shown in the Klein model. Cantellated (6,6). Q 2 tl 7 O 26 £ dt.

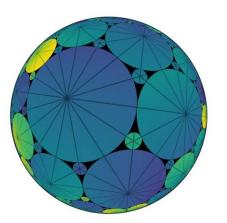


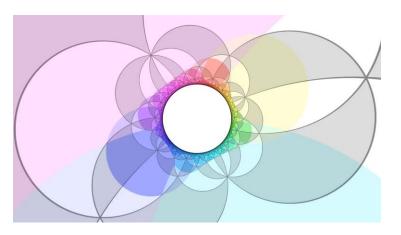
@Tilingbot

The Real Shader #1: Hyperbolic Wythoff explorer by Matt Zucker, mzucker.github.io









XPad - [D:\TilingBot\working\2019-8-05_20-59-02.xml]

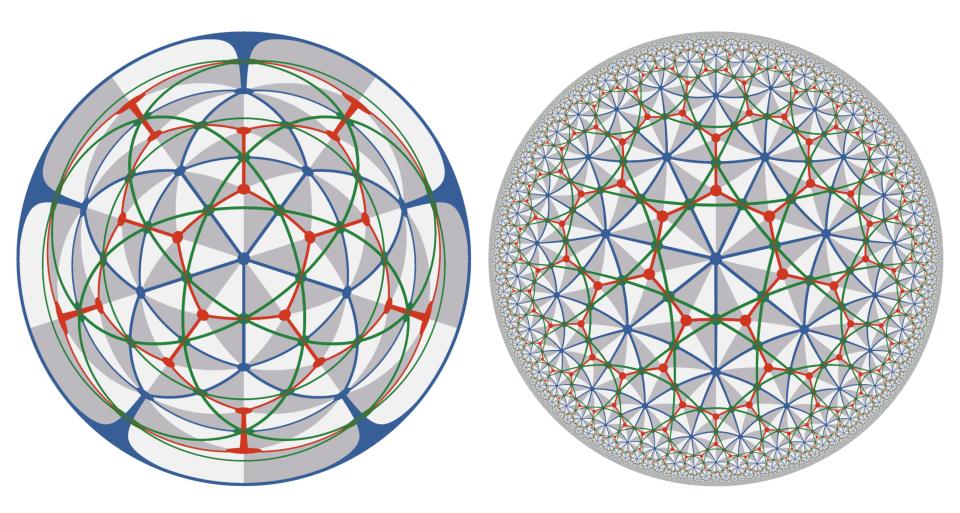
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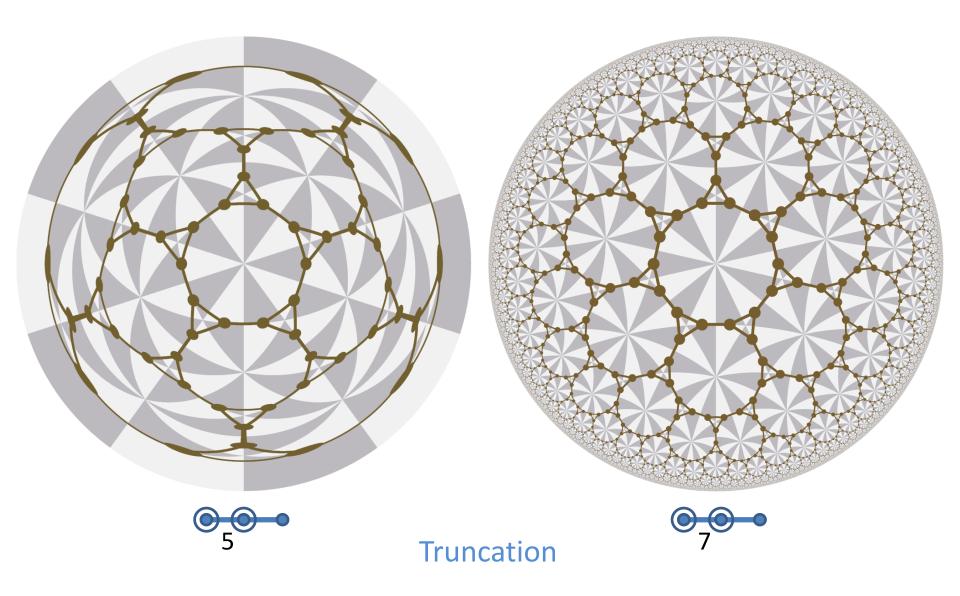
Ready

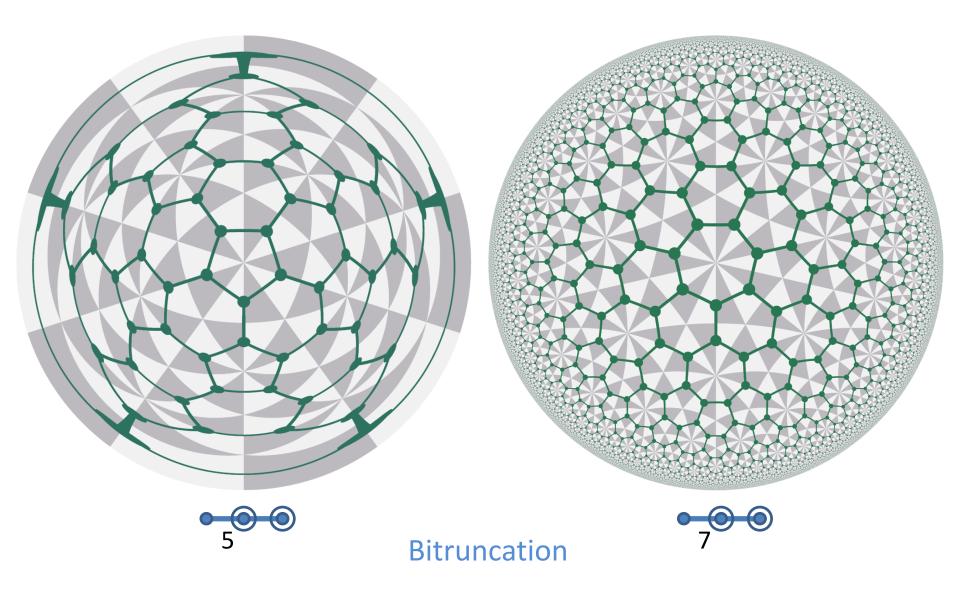
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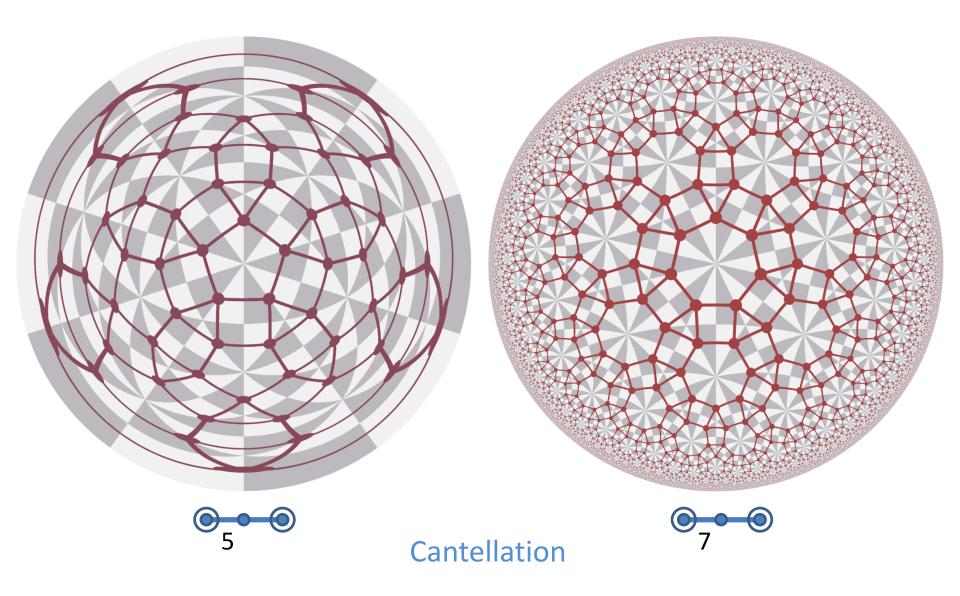
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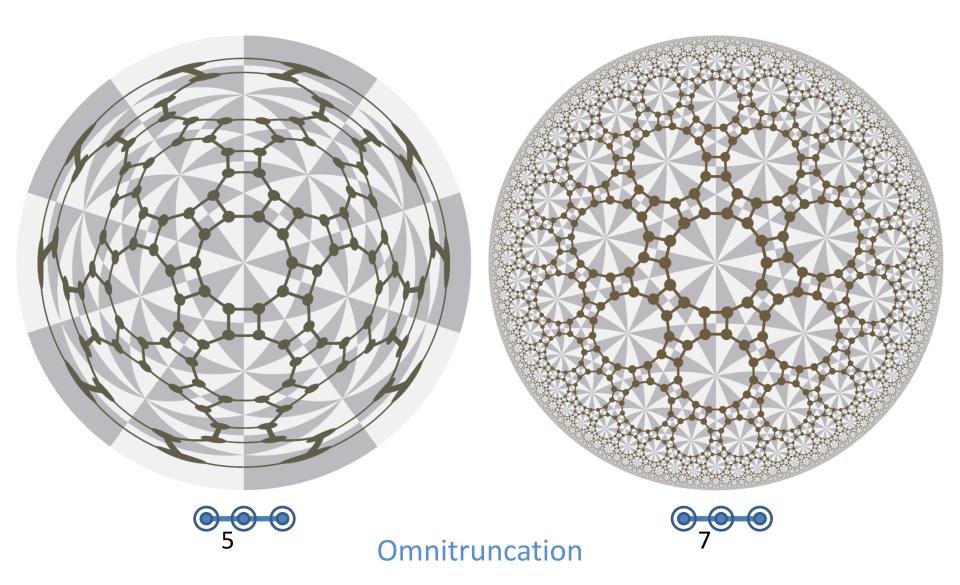
Regular and Rectified

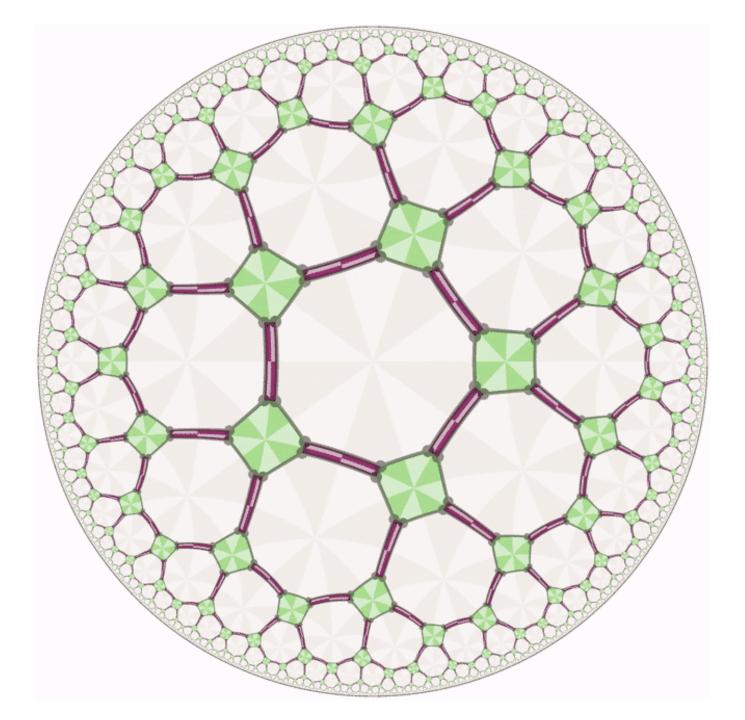




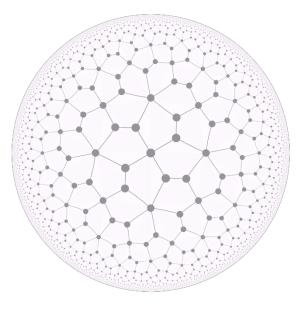


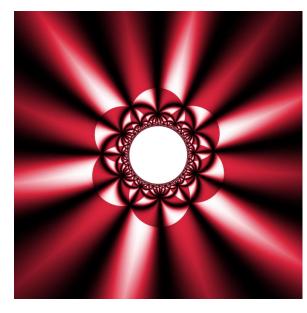


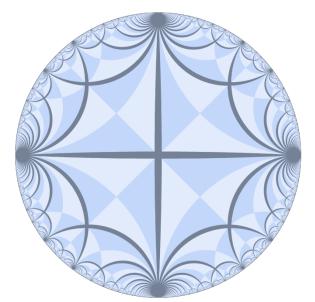


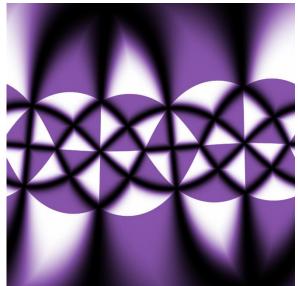


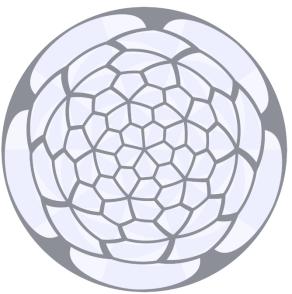
Duals to Uniform (Catalan Tilings)

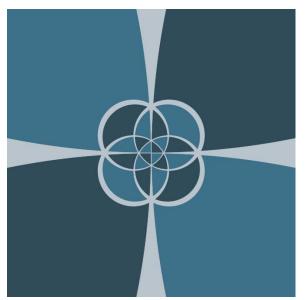




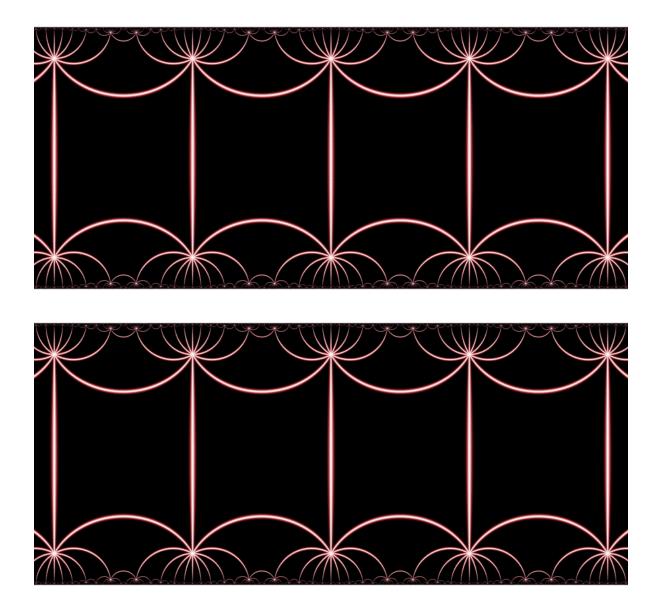




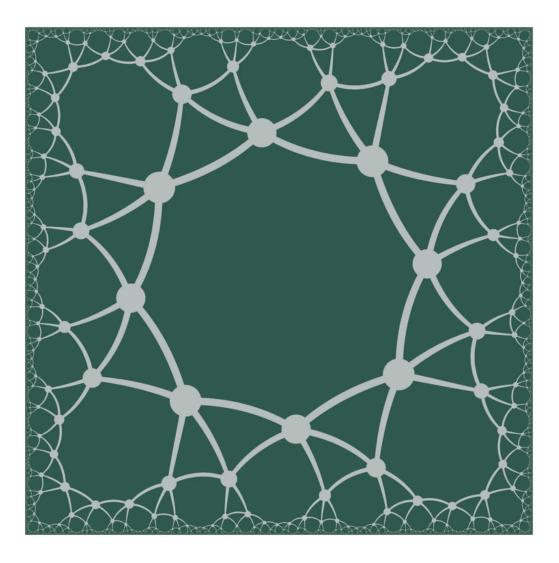




The Same But Different

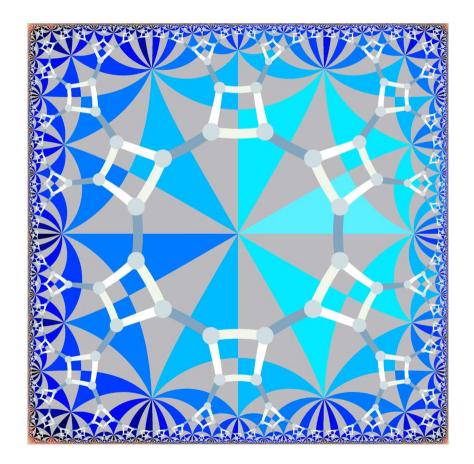


Rotating in a conformal square



Snub {8,8}

In a rotating conformal square



Omnitruncated {6,9}

2:39 🗸

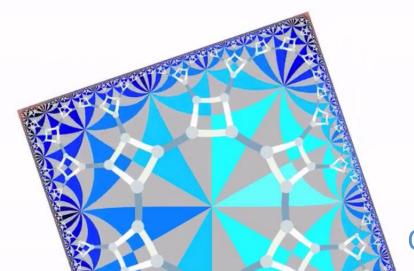
Tweet



Excellent illustration of the Koebe distortion theorem

Tiling Bot @TilingBot · 3/17/19

#Hyperbolic #tiling shown in a rotating conformal square projection. Omnitruncated {6,9}.

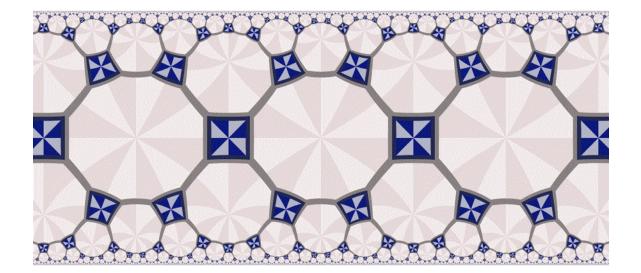


Omnitruncated {6,9}



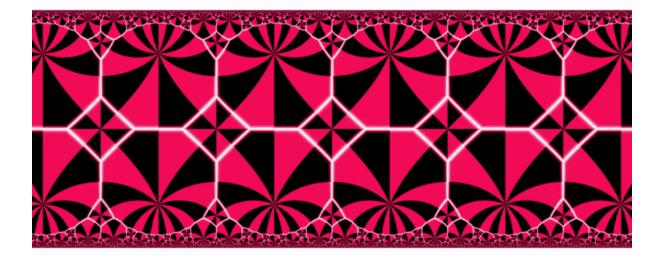
Omnitruncated {6,9}

Rotating in the band model



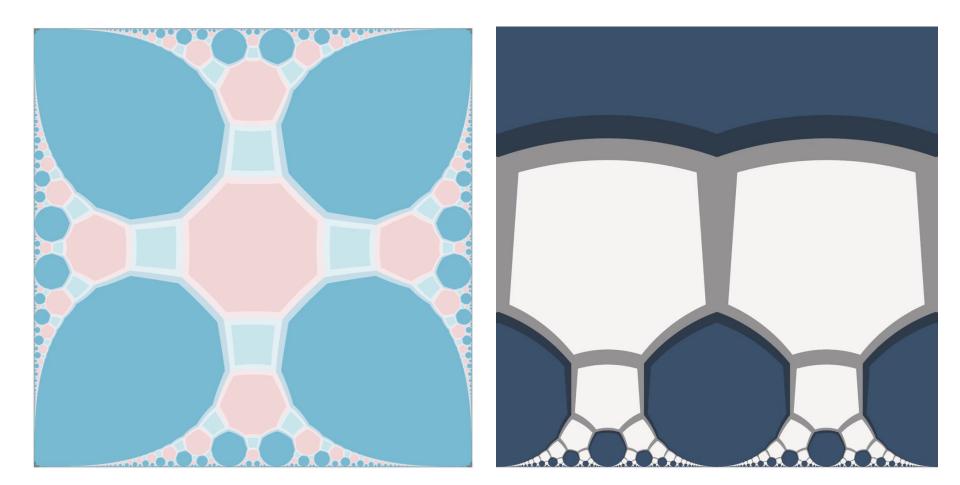
Truncated {6,4}

In a rotating band model



Truncated {8,4}

Limit Rotations



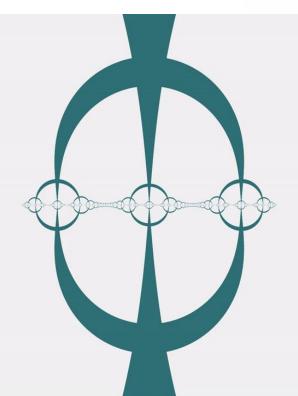
Omnitruncated {4,∞}

Truncated {3,∞}

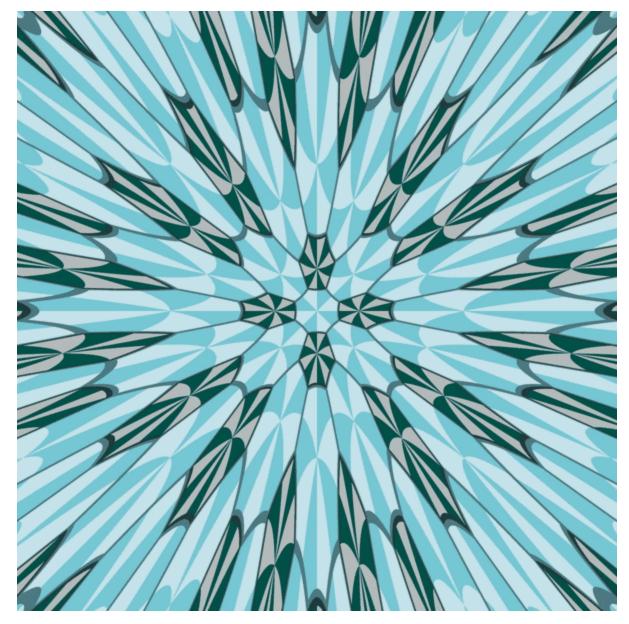
Joukowsky projection

named after Nikoli Zhukovsky

 $z = \frac{1}{2} \left(\zeta + \frac{1}{\zeta} \right)$



The best internal representation?



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